Feb. 15, 2017

Sect. 3-7

Finding Polynomial Zeros

Rational Zero "Test"

List the Possibles

Descarte's Rule of Signs

Fundamental Thrm of Algebra

Find the Roots (Zeros)

$f(x) = x^2 + 5x + 6$

Solve: $x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x + 3 = 0 \qquad x + 2 = 0$

$x = -3$ or $x = -2$

$$(x^3 - 5x^2) - (4x \cancel{+} 20) = 0$$

$$x^2(x-5) - 4(x-5) = 0$$

$$(x-5)(x^2-4) = 0$$

$$(x-5)(x+2)(x-2) = 0$$

$x - 5 = 0$ $\quad$ $x + 2 = 0$ $\quad$ $x - 2 = 0$

$x = 5$

$x = -2$ $\quad$ $x = 2$

$$x^2 + 2x - 6 = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{28}}{2} = \frac{-2 \pm 2\sqrt{7}}{2}$$

$$x = -1 \pm \sqrt{7}$$

$$x^3 - 5x^2 + 2x + 8 = 0$$

Won't factor.

Now what?

Rational Zero "Test"

Gives us a list of possible rational zeros.

$$f(x) = ax^3 + bx^2 + cx + d$$

list: $\pm \dfrac{\text{factors of } d}{\text{factors of } a}$

List the possible rational zeros

$f(x) = x^2 + 5x + 6$

list : $\pm \dfrac{\text{factors } 6}{\text{factors } 1}$

$$\pm \frac{1, 2, 3, 6}{1}$$

$$\pm \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{6}{1}$$

$$\pm 1, 2, 3, 6$$

$$f(x) = 2x^3 - 5x^2 + 4$$

list: $\pm \dfrac{\text{factor } 4}{\text{factor } 2}$

$$\pm \frac{1, 2, 4}{1, 2}$$

$$\pm \frac{1}{1}, \frac{2}{1}, \frac{4}{1}, \frac{1}{2}, \frac{2}{2}, \frac{4}{2}$$

$$\pm 1, 2, 4 \quad \frac{1}{2}$$

,

1. $x^3+5x^2-2x-15$

$\pm\dfrac{1,3,5,15}{1} \Rightarrow \pm 1, 3, 5, 15$

2. $36x^3+144x^2-x-4=0$

$\pm\dfrac{1,2,4}{1,2,3,4,6,9,12,18,36}$

$\pm 1, 2, 4, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{1}{12}, \frac{1}{18}, \frac{1}{36}$

Descarte's Rule of Signs

Gives us the # of positive real and negative real roots.

Count Sign Changes

$f(x) = \underbrace{x^2 - x}_{1} - 6$

Pos: 1 sign change

There is 1 pos. real root.

Neg: $f^{*}(x) = x^2 + x - 6$

1 s.c. 1

There is 1 neg. real root.

2. $f(x) = 36x^3 + 144x^2 - x - 4$

Neg: $f^*(x) = -36x^3 + 144x^2 + x - 4$

1

2

2 s.c.

There are 2 or 0 neg. real roots.

$$f(x) = x^3 - 2x^2 + 3x - 5$$

Pos: 3 s.c.

There are 3 or 1 pos. real roots.

Neg: $f^*(x) = -x^3 - 2x^2 - 3x - 5$

0 s.c.

There are 0 neg. real roots.

Why subtract 2 ?

Quad. Form:

$$\frac{2 \pm i\sqrt{7}}{3}$$

Complex answers ALWAYS come in pairs.

Conjugate Pairs.

$2+i$ is a root, then
$2-i$ is also an answer.

$-4-5i$ is a root, then
$-4+5i$ is also a root.

$6+\sqrt{7}$ is a root,

then $6-\sqrt{7}$ is also a root.

Find all of the roots

23. $f(x) = x^3 - 5x^2 + 2x + 8$

list: $\pm \dfrac{\text{fact. } 8}{\text{fact. } 1} = \pm 1, 2, 4, 8$

DRoS: There are 2 or 0 pos. real roots.

$f^*(x) = -x^3 - 5x^2 - 2x + 8$

There is 1 neg. real root.

Try them:

$$f(-1) = (-1)^3 - 5(-1)^2 + 2(-1) + 8$$

$$= -1 - 5 - 2 + 8 = 0$$

or

$$\begin{array}{r|rrr|r}
-1 & 1 & -5 & 2 & 8 \\
+ & \downarrow & -1 & 6 & -8 \\
\hline
 & 1 & -6 & 8 & \boxed{0}
\end{array}$$

This reduces the cubic down to a quadratic.

$$x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$x - 2 = 0$ $x - 4 = 0$

$x = 2$ $x = 4$

So the roots of

$f(x) = x^3 - 5x^2 + 2x + 8$

are $\{-1, 2, 4\}$

29. $f(x) = x^3 - 6x^2 + 13x - 10$

list : $\pm \dfrac{1, 2, 5, 10}{1} = \pm 1, 2, 5, 10$

DRoS : There are 3 or 1 pos. real roots.

There are 0 neg. real roots.

$$
\begin{array}{c|cccc}
1 & 1 & -6 & 13 & -10 \\
+ & \downarrow & 1 & -5 & 8 \\
\hline
 & 1 & -5 & 8 & -2
\end{array}
$$

$$
\begin{array}{c|cccc}
2 & 1 & -6 & 13 & -10 \\
+ & \downarrow & 2 & 8 & 10 \\
\hline
 & 1 & -4 & 5 & 0
\end{array}
$$

$$x^2 - 4x + 5$$

$$x^2 - 4x + 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 20}}{2} = \frac{4 \pm \sqrt{-4}}{2}$$

$$x = \frac{4 \pm 2i}{2} = 2 \pm i$$

5

1	5
-1	-5

So the roots of

$$f(x) = x^3 - 6x^2 + 13x - 10$$

are $\{2, 2+i, 2-i\}$

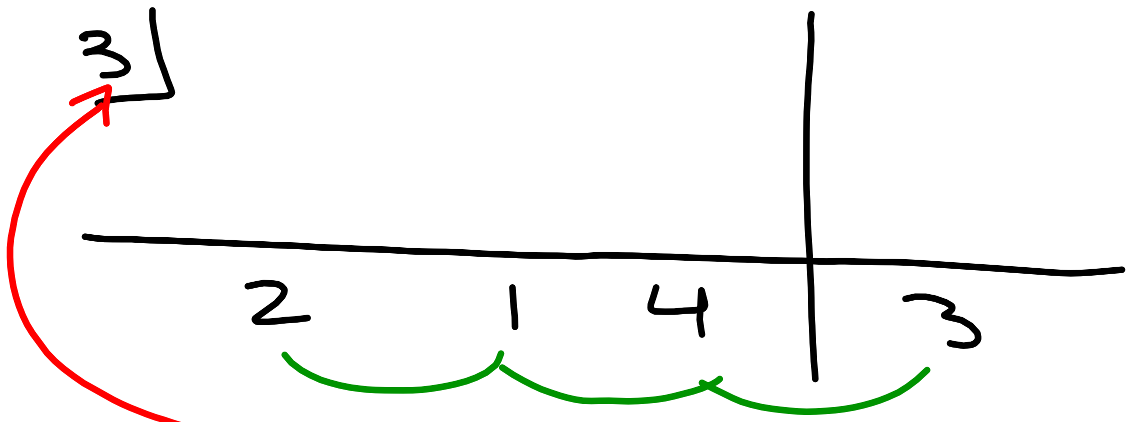
Here are a few other "tricks" to help you eliminate or find roots.

$$
\begin{array}{r|rrr|r}
1 & 1 & -6 & 13 & -10 \\
+ & \downarrow & 1 & -5 & 8 \\
\hline
 & 1 & -5 & 8 & -2
\end{array}
$$

Alternating signs.

No answers less than 1.

i.e. All answers are greater than 1.



No answers greater than 3

i.e. All answers are less than 3.

Test for $-1/1$ root of $f(x)$

Add the coeffs of $f(x)$.

$=0$? If yes, then 1 is a root.

Change the odd deg. coeffs of $f(x)$, then add.

$=0$? If yes, then -1 is a root.